Three Optical Methods for Remotely Measuring Aerosol Size Distributions

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Introduction

THE need for improved ways to measure the particle size distributions of atmospheric aerosols has been dramatized in recent years by increased public concern about air quality, increased atmospheric turbidity, possible climatic chages due to air pollution, etc. Atmospheric scientists have actually been interested in the aerosol problem for many years, for it is well known that even natural background concentrations of aerosol particles can noticeably effect the optical properties of the atmosphere.[1-4] Moreover, a limited amount of information about aerosol size distributions has already been obtained by direct sampling measurements of the type made by Junge and others.[5-9] Direct sampling techniques are, however, inherently limited insofar as measurement flexibility is concerned. Some type of remote sensing capability would therefore appear to be much more desirable, if not a necessity, for measuring aerosol size distributions throughout a large enough region of the atmosphere to conduct a meaningful aerosol surveillance program.

Optical probing techniques offer the most likely means for remotely measuring aerosol size distributions in that wavelengths in the visible and near-visible regions are short enough to be scattered rather strongly by the minute aerosol particles normally found in the atmosphere. The unique optical power sources made available by the invention of the laser also permit the use of some of the more advanced probing methods heretofore restricted to the microwave region. For example, laser radar (lidar) has already been employed in a variety of probing applications such as cloud height measurement, mapping of particulate dispersal, and detection of atmospheric structural features related to aerosol scattering inhomogeneities (see the recent lidar review by Collis,[10] for example). However, little progress has been made on the problem of determining aerosol size distributions from lidar measurements. This is not surprising in view of the fact that the size distribution information available in lidar backscatter measurements alone is naturally quite limited.[11,12] Nevertheless, it is possible to extend and augment the basic lidar technique in a way to be able to determine aerosol size distributions subject to certain physically plausible a priori assumptions. This problem has been investigated over the past three years by a laser radar research group headed by the authors at The University of Arizona, and three promising probing methods for measuring aerosol size distributions have been developed to date. These three probing methods make use of solar radiometer, monostatic lidar, and bistatic lidar sensing techniques. The theory and application of these three approaches are discussed and contrasted in the following sections of this paper.

Solar Radiometer Technique

The solar radiometer sensing technique will be considered first because the radiometer measurements may also be used to aid in the reduction of the monostatic and bistatic lidar measurements. Essentially all that is required to implement the radiometer probing technique is a means for measuring the relative flux of directly transmitted sunlight received at the Earth's surface at several wavelengths across the solar spectrum, as a function of solar zenith angle.

Neglecting the very small amount of scattered radiation

Presented as AIAA Paper 71-1057 at the Joint Conference on Sensing Environmental Pollutants, Palo Alto, Calif., November 8–10, 1971; submitted November 30, 1972; revision received July 21, 1972. The research reported in this article has been supported by the Atmospheric Sciences Section of the National Science Foundation, under Grants GA-4015 and GA-16764, and by NASA, under Grant NGR-03-003-155. The assistance of University of Arizona Lidar Research Group Personnel F. G. Fernald, G. E. Shaw, and W. P. Webster in obtaining some of the results reported here is also gratefully acknowledged.

Index category: Atmospheric, Space, and Oceanographic Sciences.

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typically within the solid angle of the solar disk, the transmitted solar energy is given by

$$F(\lambda) = F_0(\lambda)\, e^{-\tau_t(\lambda) M(\phi)} \tag{1}$$

where $F(\lambda)$ is the directly transmitted solar energy at wavelength λ received at the surface, $F_0(\lambda)$ is the corresponding value at the top of the atmosphere (at the level $\tau_t = 0$), $\tau_t(\lambda)$ is the total optical depth at wavelength λ (referenced to the zenith), and $M(\phi)$ is an airmass factor which determines the increased amount of atmospheric mass which a solar ray traverses at zenith angle ϕ relative to that traversed by a ray entering the atmosphere at the zenith. If sphericity effects are neglected (a valid approximation for zenith angles less than about 80°) and the atmosphere is assumed horizontally homogeneous, the factor $M(\phi)$ reduces to simply sec ϕ. For this case, the expression for $F(\lambda)$ may be expressed logarithmically by

$$\ln F(\lambda) = \ln F_0(\lambda) - \tau_t(\lambda) \sec\phi \tag{2}$$

and a plot of $\ln F(\lambda)$ vs $\sec\phi$ should yield a straight line of vertical intercept $F_0(\lambda)$ and slope $-\tau_t(\lambda)$ so long as the optical depth remains constant over the observation period. Figure 1 shows an example plot of this type which was determined from measurements made with a filter-wheel radiometer recently constructed at The University of Arizona for the purpose of measuring optical depths.[13] It can be seen that the plot obeys the straight line equation given by Eq. (2) rather well. This has generally proven to be the case for most of the observations made to date with the University of Arizona Solar Radiometer. The radiometer approach therefore offers what appears to be a simple and reliable means for measuring total optical depths at various wavelengths, and, as further described below, the optical depth measurements may in turn be used to infer information about the aerosol size distribution.

The total optical depth, $\tau_t(\lambda)$, is composed of a component due to the normal molecular atmospheric constituents, $\tau_r(\lambda)$, referred to as the Rayleigh optical depth, plus a component due to aerosol particles, $\tau_m(\lambda)$, referred to as the Mie optical depth.† The Rayleigh optical depth for a given wavelength is known theoretically (assuming wavelengths are selected to avoid certain absorption bands) and may be obtained from tables such as those given by Elterman.[15] Thus, the Mie optical depth may be obtained by subtracting the theoretically known Rayleigh component from the experimentally determined total optical depth to get

$$\tau_m(\lambda) = \tau_t(\lambda) - \tau_r(\lambda)$$

A basic assumption which is made at this point, partially for convenience and partially of necessity, is to assume that the relative shape of the aerosol size distribution remains constant with height. There is indeed some evidence based on measurements made by Junge[6] that such an approximation should be reasonably valid through the lower troposphere where the greatest quantity of aerosol particles are normally found. With the height independent assumption, $\tau_m(\lambda)$ may be expressed in the form

$$\tau_m(\lambda) = H \int_{a_1}^{a_2} Q_m(\lambda, a)\, da \tag{3}$$

† Aerosol particles are of the same order of size as visible wavelengths, and they therefore strongly diffract light. The only tractable scattering theory for this case is the Mie theory which applies to spherical particles. Hence, the words Mie and aerosol are normally used interchangeably when describing the optical properties of aerosols. It is known that both naturally occurring and man-made aerosol particles are not truly spherical in shape, and scattering experiments performed by Holland and Gagne[14] on randomly arrayed grossly nonspherical particles indicate that light scattering by such particles can deviate greatly from that predicted by the Mie theory, particularly with regard to the polarization characteristics of the scattered light. However, Eiden,[2] Bullrich et al.,[3] and others have used the spherical particle assumption and obtained good agreement between theory and observations in studies of scattering by atmospheric aerosols.

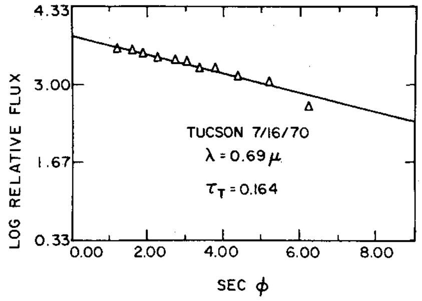

Fig. 1 Example optical depth determination. Solid line is least squares fit to the triangular data points.

and

$$H = \int_0^\infty h(z)\, dz$$

where $Q_m(\lambda, a)$ is the Mie single particle attenuation cross section at wavelength λ for a particle of radius a (and refractive index n), $f(a)$ is the normalized unit volume aerosol size distribution (normalized to the distribution at some arbitrary height), a_1 and a_2 are the lower and upper cutoff radii, respectively, of $f(a)$, $h(z)$ is a profile function which describes the relative variation in aerosol number density with height z, and H is a constant resulting from integrating $h(z)$ to the top of the atmosphere. The product of H times the integral of $f(a)$ integrated over all particle sizes equals the total number of aerosol particles in a vertical column of unit cross-sectional area which extends to the top of the atmosphere. Thus, the aerosol size distribution information contained in the Mie optical depth is actually an integrated mean which is representative of the entire (vertical) atmosphere. If $\tau_m(\lambda)$ is normalized by H, this also defines the normalized Mie volume attenuation coefficient, $\alpha_m'(\lambda)$, given by

$$\alpha_m'(\lambda) = \frac{\tau_m(\lambda)}{H} = \int_{a_1}^{a_2} Q_m(\lambda, a) f(a)\, da \tag{4}$$

The actual Mie volume attenuation coefficient at any height z, $\alpha_m(\lambda, z)$, is then given by

$$\alpha_m(\lambda, z) = \alpha_m'(\lambda)\, h(z)/H$$

The size distribution information content in the Mie optical depths may be investigated theoretically by computing the optical depth vs wavelength dependence for various model aerosol size distributions. Results from direct measurements made by Junge,[5] Clark and Whitby,[8] and others indicate that atmospheric aerosol size distributions may be modeled to a first order approximation by the Junge distribution which is given by

$$f(a) = dN/da = C\, a^{-(\nu+1)} \tag{5}$$

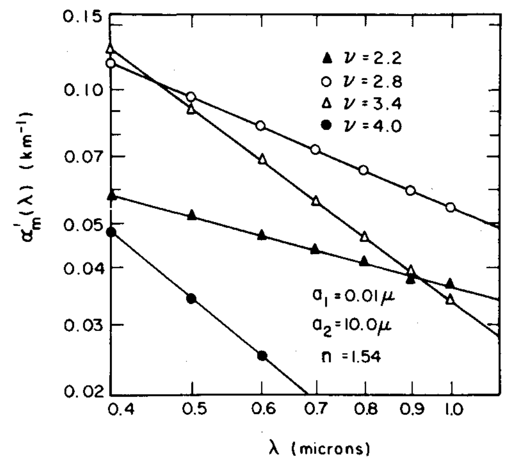

Fig. 2 Theoretically computed aerosol attenuation coefficients plotted as a function of wavelength for selected values of the Junge constant ν.

where dN is the number density of particles with radii between a and $a + da$, C is a scaling constant, and v is a distribution shaping constant. The value of v generally ranges between 2.0 and 4.0, and the distribution is typically assumed to extend from a lower cutoff radius, a_1, of about 0.01 μ to an upper cutoff radius, a_2, of about 10.0 μ. Using the Junge distribution as a model, theoretical calculations of normalized Mie optical depths as defined by Eq. (4) were made for selected wavelengths and several values of v between 2 and 4. The integral in Eq. (4) was evaluated numerically by a 300 point summation over an assumed particle size range from 0.01 to 10.0 μ. The single particle attenuation cross sections, $Q_m(\lambda, a)$, were evaluated from the Mie theory for a real particle index of refraction of $n = 1.54$,§ and the size distribution was normalized arbitrarily to correspond to a mass density of 200 μg/m^3 for each assumed value of v (see Shaw[13] for additional computational details). The wavelengths selected for computation were also chosen to correspond with the wavelengths covered by the University of Arizona Solar Radiometer. Results from some of these calculations are shown in Fig. 2 where $\alpha_m'(\lambda)$ is plotted against λ on a log-log scale for values of v equal to 2.2, 2.8, 3.4, and 4.0.

It can be seen that the slopes of the curves in Fig. 2 vary rather significantly for different values of v. This suggests the possibility of inferring the value of v for the Junge distribution which best approximates actual measurements by comparing experimentally determined Mie optical depth vs wavelength plots with the theoretical curves. However, it is not necessary to assume some a priori size distribution model such as the Junge distribution, for the optical depth measurements can be reduced to solve for an arbitrarily shaped size distribution. That is, given Mie optical depths, $\tau_m(\lambda)$, for several wavelengths, each optical depth value may be related to an integral as defined in Eq. (3), where the kernal function $Q_m(\lambda, a)$ is assumed to be known from the Mie theory, and the resulting set of linear integral equations may, at least in theory, be mathematically inverted to solve for the unknown size distribution $f(a)$. In practice, the integrals are expressed approximately by numerical summations to obtain a set of linear algebraic equations, one equation for each known value of $\tau_m(\lambda)$, and the equations are inverted numerically by matrix inversion techniques to solve for weighted solution points of the unknown distribution $f(a)$. Direct inversion of a set of equations of this type is unfeasible because the presence of even very small amounts of noise due to inherent measurement errors, quadrature errors, etc., gives rise to poor, highly oscillatory solutions which are unphysical in nature.[17] However, a constrained solution technique has been devised by Twomey[18,19] which yields smoothed solutions by applying certain physically plausible constraints to the allowed solutions. For example, one frequently employed constraint requires that the second derivative of the solution points be minimized.

Yamamoto and Tanaka[20] have recently reported on the successful application of the Twomey technique to the problem of determining aerosol size distributions from spectral attenuation measurements, and their procedure has been modified to apply to optical depth measurements.[13] Examples of a few aerosol size distributions which have been obtained by inverting optical depth measurements made in Tucson with The University of Arizona Solar Radiometer are shown in Fig. 3. The inversions were performed on Mie optical depths which were derived from radiometer measurements made at eight wavelengths. In performing the inversion calculations, it was assumed that the aerosol particles had a real refractive index of $n = 1.54$, and the Mie theory was used to evaluate all the required single particle attenuation cross sections. The error associated with the measured total optical depths was estimated to be within the $\pm 1\%$ limit for which the radiometer was designed, and the results in Fig. 3 indicate that one can expect to obtain meaningful inversions for errors of this level. Theoretical computations have also revealed that significantly improved size distribution inversions may be obtained if the optical depths are distributed over a wider wavelength range to include longer wavelengths out to two or three microns. Hence, the radiometer probing technique appears to offer a useful method for obtaining aerosol size distribution information.

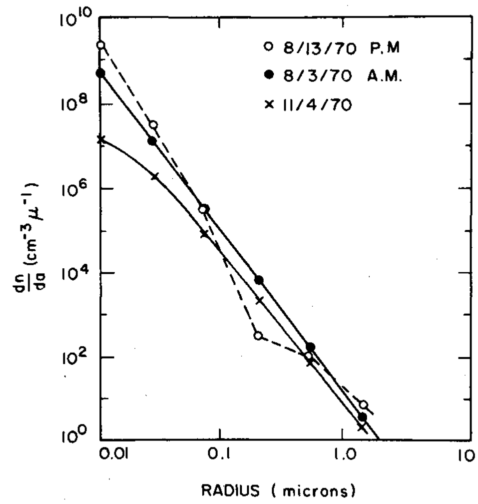

Fig. 3 Aerosol size distributions obtained by inverting optical depth depth measurements made at eight wavelengths. (results apply for $n = 1.54$).

Monostatic Lidar Technique

Monostatic (backscatter) lidar makes use of the same operating principle employed by pulsed microwave radar, except that lidar operates in the visible or near visible wavelength region. In the simplest configuration, a Q-switched laser, typically ruby or neodymium, is used to transmit light pulses into the atmosphere, and an optical telescope adjacent to the laser is used to intercept backscattered echoes from the transmitted pulses. Light collected by the telescope is focused onto a photomultiplier detector, and the detector response is displayed on an oscilloscope which is synchronized to the firing of the laser. Echoes received at any time t after firing the laser may be related to backscattering which occured at range R by $R = ct/2$ where c is the speed of light.

The received lidar response may be described quantitatively in terms of the lidar equation which is the analog of the meteoorlogical radar equation. If the molecular (Rayleigh) and the aerosol (Mie) contributions are separated, the lidar equation may be expressed in the form

$$P(R) = E_0 L R^{-2} [\beta_r(R) + \beta_m(R)] T_r^2(R)\, T_m^2(R) \tag{6}$$

where $P(R)$ is the instantaneous received power due to backscattering at range R, E_0 is the transmitted laser pulse energy, L is a constant of the lidar system, $\beta_r(R)$ is the Rayleigh volume backscatter coefficient at range R and $\beta_m(R)$ is the corresponding Mie term, and $T_r^2(R)$ and $T_m^2(R)$ are the Rayleigh and Mie transmission factors, respectively, for transmission to and from range R. The transmission factors may be related to their respective volume attenuation coefficients by

$$T_r^2(R) = \exp\left[-2\int_0^R \alpha_r(r)\, dr\right] \tag{7a}$$

§ The aerosol refractive index is assumed to be equal to a real index of $n = 1.54$ for all work presented in this paper. This value is typical of silicates, and one would assume that natural aerosols found in desert regions around Tucson would indeed have a large component of silicates. In addition, $n = 1.54$ is quite close to the refractive indices of ammonium sulfate and sodium chloride, and these materials are also common constituents found in atmospheric aerosols.[6] Finally, results of direct experimental measurements made by Hänel[16] indicate that an index of about $n = 1.54$ is apprently quite typical of a large class of atmospheric aerosols.

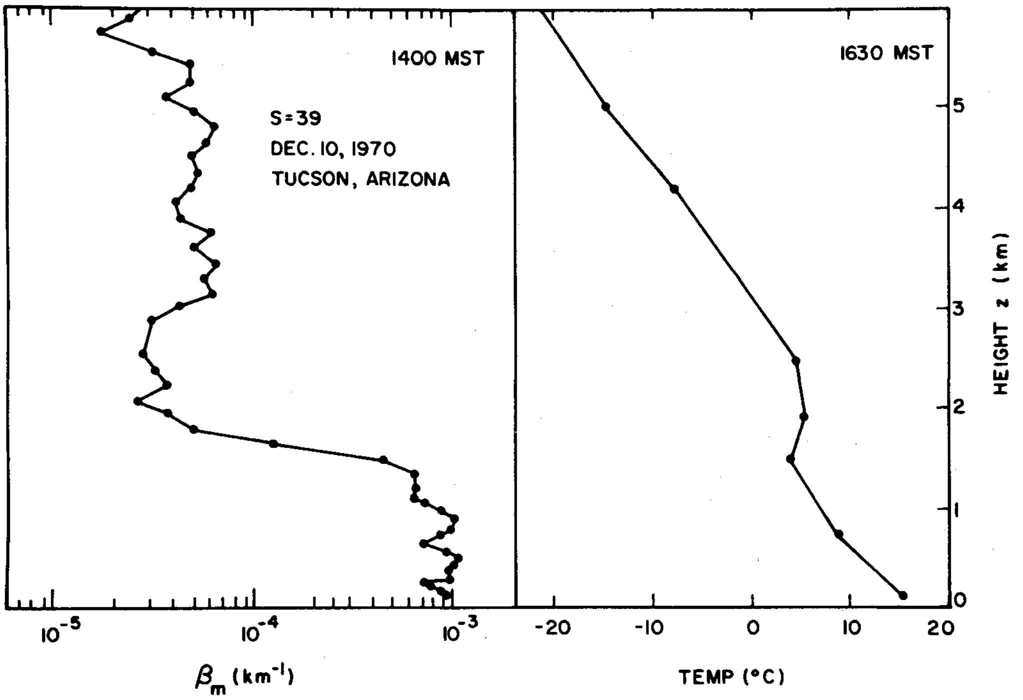

Fig. 4 Aerosol backscatter profile determined from lidar measurements and an accompanying radiosonde temperature profile.

and

$$T_m^2(R) = \exp\left[-2\int_0^R \alpha_m(r)\,dr\right] \tag{7b}$$

where $\alpha_r(r)$ is the Rayleigh volume attenuation coefficient at any intervening distance r between zero and R and $\alpha_m(r)$ is the corresponding Mie volume attenuation coefficient. The Rayleigh terms $\beta_r(R)$ and $T_r^2(R)$ or $\alpha_r(r)$ are theoretically calcuable as in the case of the Rayleigh optical depth, so the Mie terms $\beta_m(R)$ and $T_m^2(R)$ or $\alpha_m(r)$ are the only remaining unknowns. However, since the lidar response provides only one "measurement" at any range (i.e., for any value of r or R), there are still too many unknowns to be determined from the lidar response alone. This is the basic reason why the monostatic lidar technique has been largely unsuccessful as a means for providing quantitative information about aerosol size distributions. Nevertheless, as outlined below, it is indeed possible to provide the necessary auxiliary information which will permit aerosol size distributions to be inferred from lidar measurements.

As a first step, it is assumed that the relative shape of the aerosol size distribution remains constant with height and that the atmosphere is horizontally homogeneous. With these assumptions, the ratio of $\alpha_m(r)$ divided by $\beta_m(r)$ will be constant, referred to here as S, for any range r (vertical or slant path) so long as the aerosol refractive index is constant throughout the probing region. The constant S may be related to the aerosol size distribution, $f(a)$, by

$$S = \frac{\alpha_m}{\beta_m} = \int_{a_1}^{a_2} Q_m(a,\lambda_l) f(a)\,da \bigg/ \int_{a_1}^{a_2} B_m(a,\lambda_l) f(a)\,da \tag{8}$$

where $Q_m(a, \lambda_l)$ is the Mie single particle attenuation cross-section at the Lidar wavelength λ_l for a particle of radius a (and refractive index n), $B_m(a, \lambda_l)$ is the corresponding Mie single particle backscatter cross section, and $f(a)$, a_1, and a_2 have been previously defined in Eq. (3). If S were known, one could solve Eqs. (6) and (7) for the terms $\beta_m(R)$ and $T_m^2(R)$ or $\alpha_m(r)$ (see Barrett and Ben-Dov,[11] for example). However, S is more likely to be treated as an unknown to be determined because S will vary with the shape of the aerosol size distribution, and the distribution would generally be expected to vary from day to day.

The Mie transmission factor, $T_m^2(R)$, is another term which can be used conveniently to reduce the lidar measurements, and it is somewhat easier to provide information about this term than S. For example, auxiliary solar radiometer measurements could be used to determine the Mie optical depth, τ_m, at the lidar wavelength, and the Mie transmission factor to a height Z_r assumed to be above effectively all of the aerosols could be approximated by

$$T_m^2(Z_r) = e^{-2\tau_m} \tag{9}$$

In addition, taking a cue from the radiometer technique, one could make vertical and slant path lidar soundings to infer partial optical depths between ground level and any height Z. For example, a slant range response $P(R)$ taken at an angle φ with respect to the vertical divided by a vertical response $P(Z)$ may be reduced to

$$P(R)/P(Z) = \sec^{-2}\phi\, e^{\tau'(Z)[1-\sec\phi]} \tag{10}$$

and

$$\tau'(Z) = \int_0^Z [\alpha_r(z) + \alpha_m(z)]\,dz$$

where R is related to vertical height Z by $R = Z \sec \phi$, $\tau'(Z)$ is the partial optical depth between ground level and height Z, and α_r and α_m are the previously defined Rayleigh and Mie volume attenuation coefficients. The theoretically known Rayleigh contribution to $\tau'(Z)$ may be subtracted to get the Mie contribution which in turn by be used to determine the Mie transmission factor, $T_m^2(Z)$.

An efficient analysis scheme has recently been devished by Fernald et al.[21] which permits Eqs. (6) and (7) to be recast so that S and $T_m^2(Z)$ may be related in a closed form solution given by

$$T_m^2(Z) = \exp\left[2S\int_0^Z \beta_\gamma(z)\,dz\right] \times \left\{1 - \int_0^Z \frac{2SP(z)z^2 \exp\left[-2S\int_0^z \beta_\gamma(z')\,dz'\right]}{E_0 L T_r^2(z)}\right\} \tag{11}$$

where all terms have been previously defined and $T_m^2(Z)$ and S are the only terms which are not known either experimentally or theoretically. Hence, if $T_m^2(Z)$ is determined by means such as those suggested in connection with Eq. (9) or (10), one may solve for S in Eq. (11), and S may then be used with Eqs. (6), (7), and (11), to solve for $\beta_m(z)$ or $\alpha_m(z)$ at any height z between zero and Z. This procedure has been used to reduce monostatic lidar measurements made with The University of Arizona Bistatic-Monostatic Lidar.[22] An example profile of $\beta_m(z)$ inferred from one set of lidar measurements is shown in Fig. 4

along with a radiosonde temperature profile which was collected shortly after the lidar soundings. The backscatter profile is in general agreement with the thermal stability conditions indicated by the temperature profile as is evidenced by the expected sharp drop in $\beta_m(z)$ at about 1.5 km due, presumably, to the capping effect of the temperature inversion at the same height. A value of $S = 39$ was inferred for this backscatter profile, although subsequent observations indicate this value is very likely too large, probably due to a calibration error in determining the lidar system constant L. Nevertheless, this example profile suffices to show the type of information which can be extracted from monostatic lidar responses. Additional discussion concerning the problem of lidar system calibration is given by Fernald et al.[21]

As can be seen from Eq. (8) given earlier, the value of S must depend in part on the relative shape of the aerosol size distribution. Hence, with certain assumptions, it should be possible to relate the experimentally determined value of S to the unknown constants of an assumed aerosol size distribution model. The Junge model given in Eq. (5) is one obvious choice in that the model has only one shaping constant, v, which could be related on a one-to-one basis to S for a specified value of particle index of refraction. Once v is determined, the remaining Junge constant C (and hence number density) could be calculated for any height z from $\beta_m(z)$. Thus, at least limited aerosol size distribution information may be obtained with the monostatic lidar technique by introducing the proper auxiliary information, namely, $T_m^2(Z)$ at one height, and this piece of information may be determined from solar radiometer or slant path lidar measurements.

Bistatic Lidar Technique

Bistatic lidar operation parallels that of monostatic lidar, except that the receiver is placed a large distance from the transmitter so that angular scattering measurements can be made as indicated in Fig. 5. From the geometry of the bistatic configuration, it can be seen that different scattering angles, θ, may be selected by varying the transmitter and receiver pointing angles α_1 and α_2 (i.e., $\theta = \alpha_1 + \alpha_2$). Thus, the advantage of bistatic lidar over monostatic lidar is that scattering measurements may be made at several scattering angles rather than just one, thereby providing additional pieces of information which, at least in principle, can be used to infer more information about the aerosol size distribution.

Following the monostatic approach, bistatic lidar signals may be described in terms of a bistatic lidar equation.[23] Since the intensity and polarization of angularly scattered light are both influenced by the shape of the aerosol size distribution,[2] it is useful to express the bistatic lidar equation in a form which readily yields both intensity and polarization information. This may be conveniently done by resolving the received bistatic response into Stokes parameter components or some other equivalent representation (see Chandrasekhar,[24] or van de Hulst[25] for a discussion on Stokes parameters). Using the Stokes parameter formulation, the ith Stokes parameter of the scattered radiation collected by the bistatic receiver, $P_i^{(s)}$, may be related to the jth Stokes parameter of the transmitted laser pulse at the receiver, $P_j^{(t)}$, by

$$P_i^{(s)}(\theta, R_2) = \frac{L T_1 T_2}{R_2^2 \sin^2 \theta/2} F_{ij}(\theta) P_j^{(t)} \qquad (12)$$

$$i = l, r, u, \text{ or } v \qquad j = l, r, u, \text{ or } v$$

where $F_{ij}(\theta)$¶ is the ijth element of the unit volume scattering matrix which characterizes the scattering properties of the bistatic common volume for scattering through angle θ, L is a constant of the lidar system, R_2 is the distance from the receiver to the center of the bistatic common volume, and T_1 and T_2 are the one-way transmission factors to the bistatic common

¶ The wavelength dependence of $F_{ij}(\theta)$ is surpressed as wavelength is not used as a variable in this case.

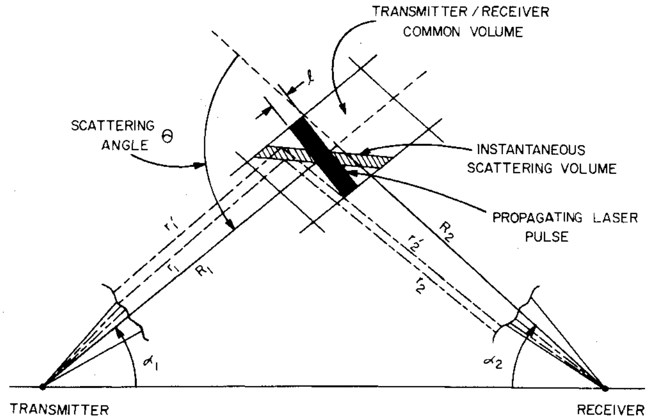

Fig. 5 Vertical plan view of bistatic lidar configuration.

volume from the transmitter and receiver, respectively. The matrix elements $F_{ij}(\theta)$ have the dimensions of cross section per unit volume per steradian, and the Stokes parameters, $P_j^{(t)}$ and $P_i^{(s)}$, are in units of power. As noted with Eq. (12), the Stokes parameter subscripts i and j each range over l, r, u, and v. The l and r subscripts identify polarized components whose electric field vectors are parallel and perpendicular to the scattering plane (plane determined by the transmitter and receiver axes), whereas the u and v subscripts identify polarization components which are related to the plane of polarization and ellipticity, respectively, of the wave.

The molecular (Rayleigh) and aerosol (Mie) scattering characteristics of the bistatic common volume probed by the lidar may be separated by resolving the unit volume scattering matrix elements, $F_{ij}(\theta)$, into Rayleigh and Mie components $F_{ij_r}(\theta)$ and $F_{ij_m}(\theta)$, respectively, so that $F_{ij}(\theta) = F_{ij_r}(\theta) + F_{ij_m}(\theta)$. The Rayleigh component is known theoretically, and the Mie component may be related to the aerosol size distribution, $f(a)$, by

$$F_{ij_m}(\theta) = \int_{a_1}^{a_2} A_{ij}(\theta, a, \lambda_l) f(a)\, da \qquad (13)$$

where $A_{ij}(\theta, a, \lambda_l)$ is the ijth Mie single particle scattering matrix element, at the lidar wavelength λ_l, for scattering through angle θ by a particle of radius a (and refractive index n), and $f(a)$, a_1, and a_2 have been previously defined in Eq. (3).

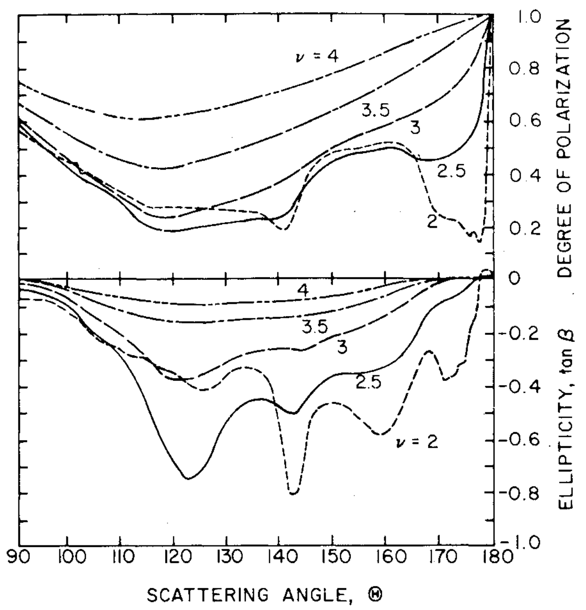

Fig. 6 Degree of polarization and ellipticity vs scattering angle θ for selected values of v ($\lambda = 0.6943\mu$ and $n = 1.54$).

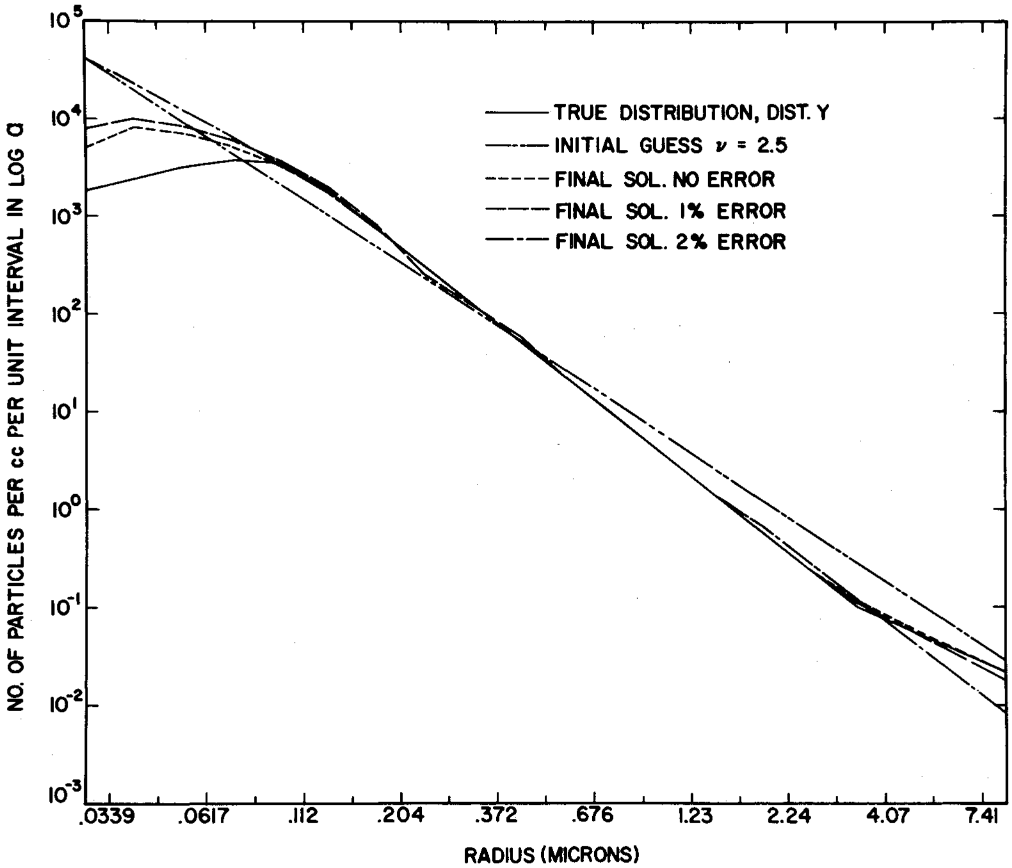

Fig. 7 Theoretical aerosol size distributions obtained by inverting simulated aerosol scattering measurements (results apply for $\lambda = 0.6943\mu$ and $n = 1.54$).

In order to investigate the sensitivity of angular scattering measurements to the shape of the aerosol size distribution, the Junge size distribution model was assumed, and the Mie matrix elements $F_{ij_m}(\theta)$ were computed for different values of θ and the Junge constant v, by numerically evaluating the integral in Eq. (13) for a real particle index of refraction of $n = 1.54$. The computed values of $F_{ij_m}(\theta)$ were used in Eq. (12) to model the aerosol contribution to the scattered Stokes parameter components, and these values were used in turn to calculate various polarization parameters of the modeled aerosol scattering contribution. All calculations were made for the case where the transmitted pulse was assumed to be linearly polarized at 45° to the scattering plane. This corresponds to the transmitter characteristics of The University of Arizona Bistatic-Monostatic Lidar. An example result from these calculations is shown in Fig. 6 where the degree of polarization and ellipticity are plotted as a function of scattering angle for five values of v. The interesting features in both sets of curves are the regions where there is a strong single valued dependence on v such as the intervals between about 165° to 175° for the degree of polarization and between about 140° to 160° for the ellipticity.

The curves in Fig. 6 indicate it should be possible to estimate the value of the Junge constant v which best approximates the aerosol size distribution by analyzing the characteristics of the polarization parameters of the aerosol scattering contribution for appropriate scattering angles. In fact, it has been demonstrated through the use of optimization techniques that it is indeed possible to accurately estimate the value of v from simulated measurements of various aerosol polarization parameters,[26] and preliminary bistatic measurements have also been reduced to infer the value of v which best approximates the measurements.[23] In addition, as in the case with multiwavelength optical depth measurements, it is possible to invert angular scattering measurements made at several scattering angles to solve for an arbitrarily shaped size distribution. The authors have recently applied the Twomey inversion technique to this problem, and an inversion routine has been developed to determine the aerosol size distribution by inverting the Stokes parameter components of the received bistatic signals for five scattering angles.[27] Example particle size distributions obtained by inverting simulated bistatic scattering measurements with zero, one, and two percent error are shown in Fig. 7. All inversions assume that the aerosol particles are characterized by a real index of refraction of $n = 1.54$, and the simulated measurements were also generated on this basis. The true distribution (solid line) was generated by modifying a Junge size distribution at the lower and upper end of the particle size range, and a Junge distribution with $v = 2.5$ was used as an initial guess. It can be seen that the inverted solution for even the two percent error case is a good approximation of the true distribution over most of the particle size range. Inversions performed with initial guesses much greater in error than the case considered here also have been found to yield good results. Thus, using the proper data analysis methods, it appears that the bistatic lidar technique may be used to obtain reasonably detailed aerosol size distribution information so long as the scattering measurements are no more than a few percent in error. Bistatic experiments are now being performed to obtain sufficiently accurate measurements so that meaningful inversions can be performed.

Comparisons and Conclusions

Each of the three probing techniques described in this paper for measuring aerosol distributions has certain advantages and disadvantages, and all of the techniques require that certain a priori assumptions be made in order to obtain size distribution information. For example, the aerosol index of refraction is assumed to be known in all cases. The aerosol particles are also assumed to be spherical so that the Mie scattering theory may be applied. As previously noted, these two assumptions can be made with reasonable confidence in many cases, and they should therefore not necessarily be judged as serious deficiencies.

The solar radiometer has a significant economic advantage in that a simple multiwavelength instrument may be constructed for a few thousand dollars. This indicates that it may be feasible to set up a network of solar radiometers to investigate the optical depth of the atmosphere on a large-scale basis. Use of the solar radiometer is obviously restricted to regions of the world which are relatively cloud free because the effect of clouds completely obscures the aerosol optical depth contribution. In order to make accurate optical depth determinations, the optical depth of the atmosphere must remain constant for a period of time of about an hour or more so that the radiometer can scan a sufficiently wide range of zenith angles. The atmosphere must also be horizontally homogeneous throughout the region scanned by the radiometer. As such, radiometer operation is more or less limited to situations where meteorological conditions are not changing very rapidly, and the radiometer location should be in a region which is free of strong localized aerosol sources that might cause horizontal inhomogeneities. Since radiometer optical depth measurements include the integrated effect of the entire vertical atmosphere, size distribution inferences made from such measurements provide no information about the vertical distribution of aerosols. Nevertheless, the type of information provided by the radiometer could be quite useful in trying to assess the effect of aerosols on radiative transfer of solar energy through the atmosphere.

Vertical monostatic lidar measurements alone do not provide sufficient information to infer the unknown constants of even a simple two-parameter aerosol size distribution model such as the Junge model unless certain auxiliary information is also provided. However, the necessary auxiliary information may be obtained from attenuation measurements made by a solar radiometer or by lidar slant path observations. Lidar does have the advantage of providing an instantaneous profile each time the laser is fired, and lidar may be readily pointed in different directions to investigate the spatial structure of aerosol inhomogeneities. Thus, in contrast to the solar radiometer, lidar can provide fairly detailed information about the vertical distribution of atmospheric aerosols. The rapid rate at which profiles may be acquired with lidar is also useful for detecting temporal variations in the spatial aerosol distribution. On the other hand, lidar measurements supplemented with auxiliary attenuation information still provide only limited information, at best, about the relative shape of the aerosol size distribution. The lidar and solar radiometer techniques are therefore somewhat complementary, and it would seem worthwhile to incorporate both methods. Indeed, such a composite probing technique would appear to offer a probing capability which is significantly better than the simple sum of the capabilities of the individual probing methods.

Turning finally to the bistatic lidar approach, it can be said that bistatic lidar has some of the spatial probing features of monostatic lidar as well as providing the capability for determining fairly detailed information about the relative shape of the aerosol size distribution. Bistatic lidar does provide a means for probing selected regions of the atmosphere, but it is less flexible than monostatic lidar in the sense that the bistatic receiver and transmitter must be redirected each time a different region is to be probed. Thus, bistatic lidar does not give an instantaneous profile, and a finite amount of time is required to make scattering measurements at several scattering angles. Temporal variations in aerosol concentrations are therefore a potential problem in trying to obtain a consistent set of angular scattering measurements which can be used for size distribution inversions. However, all things considered, the bistatic lidar technique does appear to offer the most complete approach for measuring aerosol size distributions of the three methods described in this paper.

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